Half Lives

This worksheet will cover the topic of half lives as they pertain to chemical kinetics. Half-lives provide insight into the speed at which a reaction takes place. This worksheet will discuss half lives conceptually, as well as mathematically. It will also relate the concept of half lives to reaction order, allowing you to perform calculations surrounding time and concentration. Half lives allow chemists to study general trends in reaction kinetics and predict how reactions might behave under different conditions.

1. The half life of a reaction is 10 seconds, and has an initial concentration of 1.0M. How long will it take for the concentration to decrease to 0.5M?

2. Is the half life of a reaction ever constant? Consider the equations for the half life of zero order, first order, and second order reactions.

3. With what reaction order does the half life of a reaction decrease with increasing initial concentration?

4. How can temperature affect the half-life of a reaction? Justify your answer in terms of collision theory.

5. In a second-order reaction, the concentration of a reactant A decreases from 0.7 M to 0.2 M in 30 seconds. Calculate the half life for this reaction. (Use the equation $1/[A]t = 1/[A]_0 + kt$ to help)

6. Compare and contrast the half-lives of a first-order reaction with a rate constant of 0.02 s^{-1} and a second-order reaction with a rate constant of 0.02 $M^{-1}s^{-1}$, both starting with an initial concentration of 1.5 M. Do these comparisons stay true for the succeeding half lives?

7. A reaction has a reactant "X", with an initial concentration of 0.92M, a rate constant of 0.03, and a half life of 15.33 seconds. What order is the reaction with respect to X?

8. Challenge Question - At 8pm, the concentration of reactant A in the first order reaction $A + B \rightarrow AB$ is 2.4M. Two hours later, the concentration of reactant A is 0.0375M. Calculate the half life for this reaction.

Answer Key

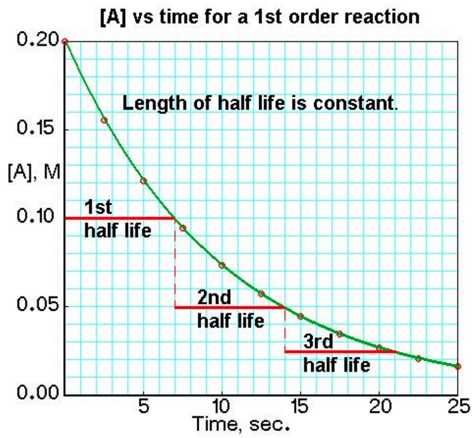
1. The half life of a reaction is 10 seconds, and has an initial concentration of 1.0M. How long will it take for the concentration to decrease to 0.5M?
Intro to Half Lives
Half lifes are the amount of time a reaction takes to reach half of its initial reactant concentration. For example, if a reactant's starting concentration is 2 M, and it takes 30 seconds to reach 1M, then the half life would be 30.

Strategy - We are given that the half life for this reaction is 10 seconds. Right away, we see that the final concentration (0.5M) is exactly half of the initial concentration (1.0M). This means it will take the duration of one half life, or **10 seconds**, to decrease to 0.5M.

2. Is the half life of a reaction ever constant? Consider the equations for the half life of zero order, first order, and second order reactions.

Intro to Half Life Equations & Reaction Order:
The formula for half lifes depends on the order of the reaction. Reaction order is always determined experimentally, and refers to the expression that relates the degree to which the rate of a reaction depends on the concentration of reactants. For example, a rate that was proportional to the concentration of a reactant $[A]^1$ would have an order of 1, while a rate proportional to $[A]^2$ would have an order of 2. Calculating half lifes can be done with the following equations, where k is the rate constant, and $[A]_0$ is the initial concentration of the reactant.
- Zero Order: $t_{1/2} = [A]_0/2k$
- First Order: $t_{1/2} = \ln(2)/k$
- Second Order: $t_{1/2} = 1/k[A]_0$

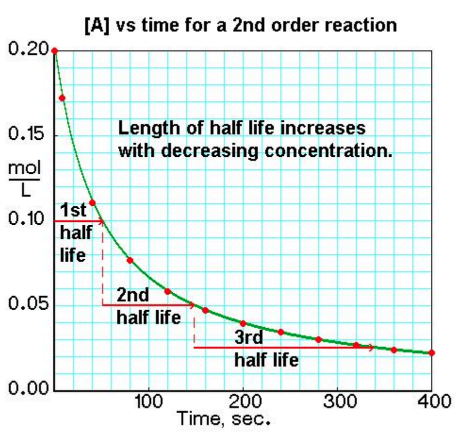
Strategy - Looking back at the equations, both zero and second order half life equations depend on the initial concentration of the reactant. However, the first order half life does not have $[A]_0$ as a term.
The half life of a first order reaction is constant, as long as the rate constant k remains the same.



Credit: Purdue University Department of Chemistry

3. With what reaction order does the half life of a reaction decrease with increasing initial concentration?

Strategy - This problem is similar to question 1. This time, reference the formula to find an equation where $t_{1/2}$ is inversely proportional to the initial concentration. This relationship is found in a **second order reaction**. $[A]_0$ is in the denominator of the expression, meaning the half life will increase with decreasing initial concentration. Graphically, the relationship can be represented as such:



Credit: Purdue University Department of Chemistry

4. How can temperature affect the half-life of a reaction? Provide a molecular explanation.

<u>Intro to Collision Theory:</u>
Collision theory, established in the early 20th century, is a foundational concept in chemistry that explains how chemical reactions occur. It states that reactions can only take place if molecules collide with sufficient energy (speed) and proper orientation.

Speed is needed to overcome the activation energy barrier, or to break the bonds between reactant molecules. Orientation is needed to ensure molecules are correctly positioned to form new bonds When molecules collide without enough energy or the correct orientation, they are called "ineffective collisions".

Strategy - Increased temperature generally makes reactions go faster by providing more energy for molecules to collide. Molecules with a higher temperature move faster, making more effective collisions possible. Therefore, a faster reaction leads to a **decrease in half life.**

5. Compare and contrast the half-lives of a first-order reaction with a rate constant of 0.02 s^{-1} and a second-order reaction with a rate constant of 0.02 $M^{-1}s^{-1}$, both starting with an initial concentration of 1.5 M.

   Strategy - Use the formulas for first and second order reactions to solve for each half life.

   First order: $t_{1/2} = ln(2) \div 0.02s^{-1} = 34.65\ seconds$

   Second order: $t_{1/2} = \frac{1}{0.02M^{-1}s^{-1} \times 1.5M} = 33.33\ seconds$

   The second order reaction will be slightly quicker to reach half its initial concentration. However, as the reaction goes on, the half life for the first order reaction will remain constant, while the second order reaction's half life will increase.

6. In a second-order reaction, the concentration of a reactant A decreases from 0.7 M to 0.2 M in 30 seconds. Calculate the half life for this reaction. (Use the equation $1/[A]_t = 1/[A]_0 + kt$ to help)

   Strategy - In a second-order reaction, the integrated rate law is given by: $1/[A]_t = 1/[A]_0 + kt$

   Given that the concentration of A decreases from 0.7 M to 0.2 M in 30 seconds, we can use these values to find the rate constant k. From there, we can solve for the half life.

   1. Initial concentration $[A]_0$ = 0.7 M
   2. Concentration after 30 seconds $[A]_{30}$ = 0.2 M
   3. Time (t) = 30 seconds

   Substitute these values into the integrated rate law, and isolate k.

   $k = (\frac{1}{0.2\ M} - \frac{1}{0.7\ M}) \div 30\ s = 0.1190\ M^{-1}s^{-1}$

   Use the value of k to find $t_{1/2}$ using the formula for second-order reactions:

   $t_{1/2} = \frac{1}{0.1190\ M^{-1}s^{-1} \times 0.7\ M}$ = **12.00 seconds**

7. A reaction has a reactant "X", with an initial concentration of 0.92M, a rate constant of 0.03, and a half life of 15.33 seconds. What order is the reaction with respect to X?
Strategy - Plug the values given into each half life equation to find which one yields and answer of 15.33 seconds. If the units of the rate constant were given, you can also determine the order of the reaction by the rate constant units.
Zero Order: $t_{1/2}$ = $\frac{0.92\ M}{2 \times 0.03}$ = 15.33 seconds
First Order: $t_{1/2}$ = $\frac{ln(2)}{0.03}$= 23.104 seconds
Second Order: $t_{1/2}$ = $\frac{1}{0.03\ \times 0.92\ M}$= 30.66 seconds
Therefore, the reaction is **zero order** with respect to reactant X.

8. Challenge Question - At 8pm, the concentration of reactant A in the first order reaction A + B → AB is 2.4M. Two hours later, the concentration of reactant A is 0.0375M. Calculate the half life for this reaction.
Strategy - In this problem, an initial concentration or rate constant is not given. However, conceptually understanding half lifes can help solve this problem. Half lifes are the amount of time a reaction takes to reach half of its initial concentration. Find the number of half lifes that have passed, and divide it by the amount of time (two hours). Half life will remain constant because the reaction is first order.

$2.4 \div 2 = 1.2$ will be the concentration after one half life has passed.

$1.2 \div 2 = 0.6 \rightarrow 0.6 \div 2 = 0.3 \rightarrow 0.3 \div 2 = 0.15 \rightarrow 0.15 \div 2 = 0.075 \rightarrow 0.075 \div 2 = 0.0375$ will be the concentration after 6 half lifes have passed.

$\frac{120\ minutes}{6\ half\ lifes}$ = **20 minutes**